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On the Canonical Line Bundles of some Finite Ramified Covering Spaces of P^n

by

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In this paper, a finite ramified covering is a generically finite morphism of complete non-singular varieties over the complex number field C. If S is a hypersurface in P^n with only normal crossings and if $S_1, S_2, \cdots, S_m$ are the irreducible components of S, one has $\pi_1(P^n - S) \cong Z\alpha_1 \oplus Z\alpha_2 \oplus \cdots \oplus Z\alpha_m/(n_1\alpha_1 + n_2\alpha_2 + \cdots + n_m\alpha_m)$, where α_i is a small loop around S_i and n_i is the degree of S_i (cf. [4]). Let $\mathbf{H}_\nu$ $(\nu = 1, 2, \cdots)$ be the subgroup of $\pi_1(P^n - S)$ generated by the elements $\nu n_1\alpha_1, \nu n_2\alpha_2, \cdots, \nu n_m\alpha_m$. We study a certain finite ramified covering $f_\nu: K_\nu \to P^n$ such that the induced morphism $K_\nu - f_\nu^{-1}(S) \to P^n - S$ is the unramified covering associated with $\mathbf{H}_\nu$. This covering was first constructed by Kawai [3], so we call it the *ν-th Kawai covering* of P^n associated with S. Our results are the following.

I) *Let S be a hypersurface in P^n with only normal crossings and let K be the ν-th Kawai covering space of P^n associated with S. Then for the canonical line bundle ω_K and the Kodaira dimension $\kappa(K)$ of K, one has the following table:*

TABLE I

m: the number of the irreducible components of S
d: the degree of S
ω_K^*: the dual line bundle of ω_K
$\kappa^{-1}(K) = \kappa(\omega_K^*, K)$

$[m > n]$

in case	ω_K
$m > \nu(d-n-1)$	ω_K^* *is ample*
$m = \nu(d-n-1)$	ω_K *is trivial*
$m < \nu(d-n-1)$	ω_K *is ample*

$[m = 1]$

in case	ω_K
$d < n+2$	ω_K^* *is ample*
$d = n+2$	ω_K *is trivial*
$d > n+2$	ω_K *is ample*

$[n \geqq m \geqq 2]$

in case			$\kappa(K)$	$\kappa^{-1}(K)$	ω_K
$m > \nu\,(d-n-1)$	$d < n+2$	$\nu < m$	$-\infty$	n	ω_K^* is ample
		$\nu = m$	$-\infty$	n	ω_K^* is semiample
		$\nu > m$	$-\infty$	n	
	$d = n+2$		$-\infty$	$m-1$	ω_K^* is semiample
	$d > n+2$		$-\infty$	$-\infty$	
$m = \nu\,(d-n-1)$	$d = n+2$		0	0	ω_K is trivial
	$d > n+2$		0	$-\infty$	
$m < \nu\,(d-n-1)$	$d = n+2$		$m-1$	$-\infty$	ω_K is semiample
	$d > n+2$	$\nu < m$	n	$-\infty$	
		$\nu = m$	n	$-\infty$	ω_K is semiample
		$\nu > m$	n	$-\infty$	ω_K is ample

II) *If* $n \geqq m \geqq 2$, *K admits a fibration* $\Psi: K \to \boldsymbol{P}^{m-1}$ *such that for a general fibre F and for* $n-m \geqq p \geqq 1$, $H^0(F, \Omega_F^p) = 0$. *Moreover one has the following table for the canonical line bundle ω_F of F*:

TABLE II

in case	ω_F
$d < n+2$	ω_F^* is ample
$d = n+2$	ω_F is trivial
$d > n+2$	ω_F is ample

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§1. Preliminaries

In this section we introduce multicyclic coverings of complete non-singular varieties, to reconstruct Kawai coverings in §2.

Suppose that there are line bundles $\mathscr{D}, \mathscr{L}_1, \mathscr{L}_2, \cdots, \mathscr{L}_m$ on a complete non-singular variety Y and, for a positive integer ν, a surjective homomorphism

$$h\colon \bigoplus_{i=1}^{m} \mathscr{L}_i^{\otimes\nu} \longrightarrow \mathscr{D}\,.$$

Let $P=\boldsymbol{P}(\bigoplus_{i=1}^{m}\mathscr{L}_i)$, $Q=\boldsymbol{P}(\bigoplus_{i=1}^{m}\mathscr{L}_i^{\otimes\nu})$ be the projective space bundles over Y associated with $\bigoplus_{i=1}^{m}\mathscr{L}_i$, $\bigoplus_{i=1}^{m}\mathscr{L}_i^{\otimes\nu}$ respectively. Then the homomorphism h defines a closed immersion $s\colon Y\to Q$, while the natural inclusion $\bigoplus_{i=1}^{m}\mathscr{L}_i^{\otimes\nu}\to S^\nu(\bigoplus_{i=1}^{m}\mathscr{L}_i)$ induces a finite morphism $\Phi\colon P\to Q$ over Y, where $S^\nu(\bigoplus_{i=1}^{m}\mathscr{L}_i)$ is the ν-th symmetric tensor power of $\bigoplus_{i=1}^{m}\mathscr{L}_i$. Let Z be the image of Y by s and put $X=\Phi^{-1}(Z)$. Then we have the following commutative diagram:

$$\begin{tikzcd} X \arrow[r, "i_X"] \arrow[d, "\phi"'] & P \arrow[d, "\Phi"'] \arrow[dr, "pr_1"] & \\ Z \arrow[r, "i_Z"] & Q \arrow[r, "pr_2"'] & Y \end{tikzcd} \tag{1.1}$$

We put $f=pr_1\circ i_X$ and call this f the *multicyclic covering* of Y associated with h. Note that f is a finite morphism. Let $\mathcal{O}_P(1)$ be the tautological line bundle of P and put $\mathcal{O}_X(1)=\mathcal{O}_P(1)\otimes\mathcal{O}_X$. We call the natural homomorphism $\tilde{h}\colon \bigoplus_{i=1}^{m}f^*\mathscr{L}_i\to\mathcal{O}_X(1)$ the *universal quotient* of the multicyclic covering f. Let $u\colon W\to Y$ be an arbitrary morphism of schemes and let $\tau\colon \bigoplus_{i=1}^{m}u^*\mathscr{L}_i\to\mathscr{E}$ be a 1-quotient on W. Then τ induces a surjective homomorphism $\bigoplus_{i=1}^{m}u^*\mathscr{L}_i^{\otimes\nu}\to\mathscr{E}^{\otimes\nu}$. If $\mathscr{E}^{\otimes\nu}$ is isomorphic to $u^*\mathscr{D}$ as a quotient line bundle of $\bigoplus_{i=1}^{m}u^*\mathscr{L}_i^{\otimes\nu}$, then it can be easily checked that there exists a unique morphism $v\colon W\to X$ over Y such that $\mathscr{E}$ is isomorphic to $v^*\mathcal{O}_X(1)$ as a quotient line bundle of $\bigoplus_{i=1}^{m}u^*\mathscr{L}_i$.

The homomorphism h induces homomorphisms of line bundles $h_i\colon \mathscr{L}_i^{\otimes\nu}\to\mathscr{D}$ ($i=1,2,\cdots,m$). If it is not a zero-homomorphism, h_i defines an effective divisor S_i such that $\mathcal{O}(-S_i)\subseteq\mathscr{L}_i^{\otimes\nu}\otimes\mathscr{D}^*$. We call the homomorphism h a *regular quotient* of $\bigoplus_{i=1}^{m}\mathscr{L}_i$ with multiplicity ν, if:

i) none of h_i's is a zero-homomorphism, and none or only one of h_i's is an isomorphism;

ii) each divisor S_i is non-singular, and the divisor $S_1+\cdots+S_m$ has only normal crossings.

If the homomorphism h is a regular quotient, then the multicyclic covering space X is also a complete non-singular variety and the universal quotient $\tilde{h}$ is also a regular quotient (cf. [1]). Let T_i be the effective divisor on X defined by the induced homomorphism $\tilde{h}_i\colon f^*\mathscr{L}_i\to\mathcal{O}_X(1)$. It is clear that $f^*S_i=\nu T_i$ for every i. Moreover we have (cf. [1])

LEMMA. *Let f, S_i, T_i be as above and let $R_1,\cdots,R_r$ $(r\geq 0)$ be non-singular effective divisors on Y such that the divisor $R_1+\cdots+R_r+S_1+\cdots+S_m$ has only normal crossings. Then:*

a) *each divisor f^*R_i on X is non-singular;*
b) *the divisor $f^*R_1+\cdots+f^*R_r+T_1+\cdots+T_m$ has only normal crossings.*

To study the canonical line bundles of Kawai covering spaces, we need the following:

PROPOSITION. *Let $h\colon \bigoplus_{i=1}^m \mathscr{L}_i^{\otimes\nu}\to\mathscr{D}$ be a regular quotient of $\bigoplus_{i=1}^m \mathscr{L}_i$ with multiplicity ν on a complete non-singular variety Y, and let $f\colon X\to Y$ be the multicyclic covering associated with h. Let $\mathcal{O}_X(1)$ be the universal quotient line bundle of f, and let ω_X, ω_Y be the canonical line bundles of X, Y respectively. Then one has*

$$\omega_X\cong f^*\omega_Y\otimes f^*(\mathscr{L}_1\otimes\cdots\otimes\mathscr{L}_m)^{\otimes 1-\nu}\otimes\mathcal{O}_X(m(\nu-1)).$$

Proof. We consider the diagram (1.1). Let $\mathscr{N}_X$ be the normal bundle of X in P, and let $\Omega_{P/Y}$ be the sheaf of relative differentials of P over Y. Then we have

$$\omega_X\cong\omega_P\otimes\det\mathscr{N}_X\cong f^*\omega_Y\otimes\det\Omega_{P/Y}\otimes\det\mathscr{N}_X. \tag{1.2}$$

Recall that $P=\boldsymbol{P}(\bigoplus_{i=1}^m \mathscr{L}_i)$. Hence we get

$$\det\Omega_{P/Y}\cong\det\left(\bigoplus_{i=1}^m pr_1^*\mathscr{L}_i\right)\otimes\mathcal{O}_P(-m). \tag{1.3}$$

Let $\mathscr{N}_Z$ be the normal bundle of Z in Q. Taking notice of the fact that $X=\Phi^{-1}(Z)$, one can check that

$$\mathscr{N}_X^*\cong\phi^*\mathscr{N}_Z^*. \tag{1.4}$$

Recall that Z is a section of $pr_2\colon Q\to Y$. Hence we have

$$\mathscr{N}_Z^*\cong\Omega_{Q/Y}\otimes\mathcal{O}_Z. \tag{1.5}$$

Since $Q=\boldsymbol{P}(\bigoplus_{i=1}^m \mathscr{L}_i^{\otimes\nu})$, it follows from (1.4) and (1.5) that

$$\det\mathscr{N}_X^*\cong\det\left(\bigoplus_{i=1}^m f^*\mathscr{L}_i^{\otimes\nu}\right)\otimes\mathcal{O}_X(-m\nu). \tag{1.6}$$

Then from (1.2), (1.3) and (1.6), we obtain the desired isomorphism. Q.E.D.

§2. The canonical line bundles of Kawai coverings

In this section, we reconstruct each Kawai covering of $\boldsymbol{P}^n$ as a sequence of multicyclic coverings and a blowing-up, and prove our results I) and II).

Construction of Kawai coverings: Let S be a hypersurface in $\boldsymbol{P}^n$ with only normal crossings. Let $S_1, S_2,\cdots,S_m$ be the irreducible components of S, and put $n_i=\deg S_i$. The divisor S_1 defines a homomorphism of line bundles $\mathcal{O}_{\boldsymbol{P}^n}\to\mathcal{O}_{\boldsymbol{P}^n}(n_1)$. From this and the identity homomorphism $\mathcal{O}_{\boldsymbol{P}^n}(n_1)\to\mathcal{O}_{\boldsymbol{P}^n}(n_1)$, we get a surjective homomorphism $\sigma_1\colon\mathcal{O}_{\boldsymbol{P}^n}\oplus\mathcal{O}_{\boldsymbol{P}^n}(n_1)\to\mathcal{O}_{\boldsymbol{P}^n}(n_1)$. The structure sheaf $\mathcal{O}_{\boldsymbol{P}^n}$ can be regarded as $\mathcal{O}_{\boldsymbol{P}^n}^{\otimes n_1}$, so that σ_1 is a regular quotient of $\mathcal{O}_{\boldsymbol{P}^n}\oplus\mathcal{O}_{\boldsymbol{P}^n}(1)$ with multiplicity n_1. Let

$\phi_1: X_1 \to \boldsymbol{P}^n$ be the multicyclic covering associated with σ_1. This is the usual cyclic covering with degree n_1 associated with S_1. Now by Lemma in § 1, we see that the divisor $\phi_1^* S_2 + \cdots + \phi_1^* S_m$ on X_1 has only normal crossings. Thus for each $i \geqq 2$, we obtain inductively a regular quotient

$$\sigma_i: \mathcal{O}_{X_{i-1}} \oplus (\phi_1 \circ \cdots \circ \phi_{i-1})^* \mathcal{O}_{\boldsymbol{P}^n}(n_i) \longrightarrow (\phi_1 \circ \cdots \circ \phi_{i-1})^* \mathcal{O}_{\boldsymbol{P}^n}(n_i)$$

of $\mathcal{O}_{X_{i-1}} \oplus (\phi_1 \circ \cdots \circ \phi_{i-1})^* \mathcal{O}_{\boldsymbol{P}^n}(1)$ with multiplicity n_i defined by the divisor $(\phi_1 \circ \cdots \circ \phi_{i-1})^* S_i$ on X_{i-1}, and the multicyclic covering $\phi_i: X_i \to X_{i-1}$ associated with σ_i. We put $K = X_m$, $\phi = \phi_1 \circ \cdots \circ \phi_m$ and $\mathcal{O}_K(1) = \phi^* \mathcal{O}_{\boldsymbol{P}^n}(1)$. For each i, the universal quotient $\mathcal{O}_{X_i} \oplus (\phi_1 \circ \cdots \circ \phi_i)^* \mathcal{O}_{\boldsymbol{P}^n}(1) \to \mathcal{O}_{X_i}(1)$ of ϕ_i induces an isomorphism $\mathcal{O}_{X_i}(1) \cong (\phi_1 \circ \cdots \circ \phi_i)^* \mathcal{O}_{\boldsymbol{P}^n}(1)$ and a homomorphism $\mathcal{O}_{X_i} \to \mathcal{O}_{X_i}(1)$, which defines an effective divisor $\tilde{S}_i$ on X_i. Put $T_i = (\phi_{i+1} \circ \cdots \circ \phi_m)^* \tilde{S}_i$ $(i = 1, \cdots, m-1)$ and $T_m = \tilde{S}_m$. Then by Lemma in § 1, each divisor T_i is non-singular and the divisor $T_1 + \cdots + T_m$ has only normal crossings. Furthermore one has $\mathcal{O}(T_i) \cong \mathcal{O}_K(1)$. Therefore each divisor T_i defines a homomorphism $\tau_i: \mathcal{O}_K \to \mathcal{O}_K(1)$. From these τ_i's we obtain a homomorphism $\tau: \mathcal{O}_K^{\oplus m} \to \mathcal{O}_K(1)$. For each $\nu \geqq 1$, we can regard τ as a homomorphism

$$\tau(\nu): (\mathcal{O}_K^{\otimes \nu})^{\oplus m} \longrightarrow \mathcal{O}_K(1).$$

In case $m > n$, the homomorphism $\tau(\nu)$ is surjective. Therefore $\tau(\nu)$ is a regular quotient of $\mathcal{O}_K^{\oplus m}$ with multiplicity ν. Let $\psi_\nu: K_\nu \to K$ be the multicyclic covering associated with $\tau(\nu)$, and put $f_\nu = \phi \circ \psi_\nu$. In case $2 \leqq m \leqq n$, let $\varepsilon: \tilde{K} \to K$ be the blowing-up of K with center $T_1 \cap \cdots \cap T_m$, and let E be the exceptional divisor of ε. Put $\mathcal{O}_{\tilde{K}}(1) = \varepsilon^* \mathcal{O}_K(1) \otimes \mathcal{O}(-E)$. Then the homomorphism $\tau(\nu)$ induces a regular quotient of $\mathcal{O}_{\tilde{K}}^{\oplus m}$ with multiplicity ν on $\tilde{K}$:

$$\tilde{\tau}(\nu): (\mathcal{O}_{\tilde{K}}^{\otimes \nu})^{\oplus m} \longrightarrow \mathcal{O}_{\tilde{K}}(1).$$

Let $\psi_\nu: K_\nu \to \tilde{K}$ be the multicyclic covering associated with $\tilde{\tau}(\nu)$, and put $f_\nu = \phi \circ \varepsilon \circ \psi_\nu$. In case $m = 1$, we put $K_\nu = K$ and $f_\nu = \phi$ for every $\nu \geqq 1$.

In any case, the morphism $f_\nu: K_\nu \to \boldsymbol{P}^n$ is a finite ramified covering such that the induced morphism $K_\nu - f_\nu^{-1}(S) \to \boldsymbol{P}^n - S$ is the unramified covering associated with the subgroup $\mathbf{H}_\nu \subset \pi_1(\boldsymbol{P}^n - S)$ (cf. [1]).

Proof of the result I): By Proposition in § 1, we have

$$\omega_K \cong \phi^* \omega_{\boldsymbol{P}^n} \otimes \mathcal{O}_K(d-m) \cong \mathcal{O}_K(d-n-1-m). \tag{2.1}$$

First we assume $2 \leqq m \leqq n$. Then we have

$$\omega_{\tilde{K}} \cong \varepsilon^* \omega_K \otimes \mathcal{O}((m-1)E) \cong \mathcal{O}_{\tilde{K}}(d-n-1-m) \otimes \mathcal{O}((d-n-2)E).$$

Put $\tilde{E} = \psi_\nu^* E$ and let $\mathcal{O}_{K_\nu}(1)$ be the universal quotient line bundle of ψ_ν. Then again by Proposition in § 1, we obtain

$$\omega_{K_\nu} \cong \psi_\nu^* \omega_{\tilde{K}} \otimes \mathcal{O}_{K_\nu}(m(\nu-1)) \cong \mathcal{O}_{K_\nu}(\nu(d-n-1)-m) \otimes \mathcal{O}((d-n-2)\tilde{E}). \tag{2.2}$$

It is clear that (a) $\mathcal{O}(E)$ is a non-torsion line bundle with $\kappa(\mathcal{O}(E), \tilde{K}) = 0$. The line bundle $\mathcal{O}_{\tilde{K}}(1)$ is a quotient of $\mathcal{O}_{\tilde{K}}^{\oplus m}$. On the other hand, since the center $T=T_1\cap\cdots\cap T_m$ of the blowing-up ε is a non-singular subvariety in K, for each closed point $t\in T$, the fibre $E_t=\varepsilon^{-1}(t)$ is isomorphic to $\boldsymbol{P}^{m-1}$ and (b) the pullback $\mathcal{O}_{\tilde{K}}(1)\otimes\mathcal{O}_{E_t}$ of $\mathcal{O}_{\tilde{K}}(1)$ to E_t is ample. Thus we see that (c) $\mathcal{O}_{\tilde{K}}(1)$ is a semiample line bundle with $\kappa(\mathcal{O}_{\tilde{K}}(1),\tilde{K})=m-1$. Recall that the line bundle $\mathcal{O}_K(1)$ is ample. Then we see that (d) the line bundle $\mathcal{O}_{\tilde{K}}(1)\otimes\mathcal{O}(E)$ is semiample and $\kappa(\mathcal{O}_{\tilde{K}}(1)\otimes\mathcal{O}(E),\tilde{K})=n$. Furthermore it follows from the fact (b) that (e) $\mathcal{O}_{\tilde{K}}(s)\otimes\mathcal{O}(tE)$ is ample if $s>t>0$. Finally, from the facts (a), (c) and (d) we obtain (f) $\kappa(\mathcal{O}_{\tilde{K}}(s)\otimes\mathcal{O}(tE),\tilde{K})=-\infty$ if either $s<0$ or $t<0$. Paying attention to the above facts (a)$\sim$(f) and the fact that $\psi_\nu^*\mathcal{O}_{\tilde{K}}(1)\cong\mathcal{O}_{K_\nu}(\nu)$, one obtains the result from (2.2). In the same manner as above, we get $\omega_{K_\nu}\cong\mathcal{O}_{K_\nu}(\nu(d-n-1)-m)$ if $m>n$. In case $m=1$, we have (2.1). In either case the result follows immediately. Q.E.D.

COROLLARY.*) *Let S be a hypersurface in $\boldsymbol{P}^n$ with only normal crossings, and suppose* $\deg S\leqq n+1$. *Then given a finite ramified covering $f\colon X\to\boldsymbol{P}^n$ which is unramified over $\boldsymbol{P}^n-S$, one has*

$$\kappa(X)=-\infty\,.$$

Proof. Regarded as a subgroup of $\pi_1(\boldsymbol{P}^n-S)$, $\pi_1(X-f^{-1}(S))$ is of finite index. So one has $\mathbf{H}_\nu\subset\pi_1(X-f^{-1}(S))$ for a suitable ν. This implies that there exists a surjective morphism $K_\nu-f_\nu^{-1}(S)\to X-f^{-1}(S)$ over $\boldsymbol{P}^n-S$, and hence a dominant rational map $K_\nu\to X$. Since $\kappa(K_\nu)=-\infty$ if $\deg S\leqq n+1$, we get $\kappa(X)=-\infty$.

Q.E.D.

Proof of the result II): Let $\Psi\colon K_\nu\to\boldsymbol{P}^{m-1}$ be the canonical morphism defined by the universal quotient $\mathcal{O}_{K_\nu}^{\oplus m}\to\mathcal{O}_{K_\nu}(1)$, and let F be a general fibre of Ψ. Then we have the following Koszul resolution of $\mathcal{O}_F$:

$$\begin{aligned}0\longrightarrow\Lambda^{m-1}(\mathcal{O}_{K_\nu}(-1)^{\oplus m-1})\longrightarrow\cdots\longrightarrow\Lambda^2(\mathcal{O}_{K_\nu}(-1)^{\oplus m-1})\\ \longrightarrow\Lambda^1(\mathcal{O}_{K_\nu}(-1)^{\oplus m-1})\longrightarrow\mathcal{O}_{K_\nu}\longrightarrow\mathcal{O}_F\longrightarrow0\,.\end{aligned}$$

In the same manner as in the proof of theorem in [1], one can prove that

$$R^p(f_\nu)_*(\mathcal{O}_{K_\nu}(-t))=0\qquad\text{for}\quad p\geqq1$$

and for $0\leqq t\leqq m-1$, and that

$$(f_\nu)_*\mathcal{O}_{K_\nu}(-t)\cong\bigoplus_{\substack{(j_1,\cdots,j_m)\in J\\(k_1,\cdots,k_m,k)\in I(-t)}}\mathcal{O}_{\boldsymbol{P}^n}(k-j_1-\cdots-j_m)$$

for $0\leqq t\leqq m-1$, where $I(-t)=\{(k_1,\cdots,k_m,k)\in\boldsymbol{Z}^{m+1}\,|\,k_1+\cdots+k_m+\nu k=-t;\ 0\leqq k_i<\nu$ for $i=1,\cdots,m\}$ and $J=\{(j_1,\cdots,j_m)\in\boldsymbol{Z}^m\,|\,0\leqq j_i<n_i$ for $i=1,\cdots,m\}$. Hence we obtain

*) The author was taught by Prof. S. Iitaka that this can be obtained from the theory of logarithmic Kodaira dimension.

$$H^p(K_\nu, \Lambda^t(\mathcal{O}_{K_\nu}(-1)^{\oplus m+1}))=0 \qquad \text{for} \quad n-1\geqq p\geqq 1$$

and for $0\leqq t\leqq m-1$. Then from the above resolution of $\mathcal{O}_F$ we get

$$H^p(F, \mathcal{O}_F)=0 \qquad \text{for} \quad n-m\geqq p\geqq 1$$

and

$$H^0(F, \mathcal{O}_F)\cong \boldsymbol{C}.$$

Thus we see that F is connected and hence that Ψ is a fibration. Furthermore since $H^0(F, \Omega_F^p)\cong H^p(F, \mathcal{O}_F)$ for $n-m+1\geqq p\geqq 0$, we have $H^0(F, \Omega_F^p)=0$ for $n-m\geqq p\geqq 1$. Now it follows from (2.2) that

$$\omega_F\cong \omega_{K_\nu}\otimes\mathcal{O}_F\cong\mathcal{O}((d-n-2)\tilde{E})\otimes\mathcal{O}_F.$$

Since it is the pullback of an ample line bundle, say, $\mathcal{O}_{K_\nu}(2)\otimes\mathcal{O}(\tilde{E})$ on K_ν to F, the line bundle $\mathcal{O}(\tilde{E})\otimes\mathcal{O}_F$ on F is ample. Hence we obtain Table II. Q.E.D.

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